Bradley University

Mechanical Engineering

ME 273

Memo to: Hon. Professor Roos

From: Zachary Abboud

Date: 12/14/22

RE: Model of Saturn V Launch

Objectives

In this experiment, a computational model was built to model the launch of the Saturn V rocket. The model was developed based on a system of three thrusters used to propel the rocket into the moon's orbit. The model began first by assessing the total force of propulsion for the rocket due the to the exhaust force from the burning and thrust where the equation can be seen below in Eq. (1).

$$m_r a \approx \sum F_{ext} + T \tag{1}$$

Where m_r is the mass of the rocket, a is the acceleration, F_{ext} is the force of the exhaust, and T is the thrust force. After the force was properly defined and successfully incorporated into the model the next component was defining the acceleration for the rocket which can be seen in Eq. (2) below.

$$a(t) \approx -\frac{GM_E}{(R_E + y(t))^2} + \frac{T}{m_r(t)} \tag{2}$$

Where a is the acceleration, G is the gravitational constant, M_E is the mass of the earth, R_E is the radius of the Earth, y is the position, T is the thrust, and m_r is the mass of the rocket. The time-dependent mass of the rocket function was then defined as the mass of the rocket changes with the jettisoning of the thrusters and the burning of the fuel and can be found below in Eq. (3).

$$m_r(t + \Delta t) = m_r(t) - \frac{T}{v_e}\Delta t \tag{3}$$

Where m_r is the mass of the rocket, t is the time, Δt is the change in time, T is the thrust, and v_e is the exhaust velocity. The velocity change with time was then defined with a simple Euler's formula and can be found below in Eq. (4).

$$v(t + \Delta t) \approx v(t) + a(t)\Delta t \tag{4}$$

Where v is the velocity, t is the time, Δt is the change in time, and a is the acceleration. The position was then defined as well with a simple Euler's and can be found below in Eq. (5).

$$y(t + \Delta t) \approx y(t) + v(t)\Delta t \tag{5}$$

Where y is the position, t is the time, Δt is the change in time, and v is the velocity. Once all the equations had been developed and assigned to each of the stages the model was fully completed, and the results were plotted and analyzed.

Results

A computational model in MATLAB was developed to model the Saturn V rocket's mission to orbit the moon. The model was developed based on given parameters by NASA for the according rocket. A model was developed to properly assess the acceleration, velocity, and altitude traveled by the rocket. The model was initially developed by defining initial parameters for the rocket. The radius & mass of the earth, gravitational constant, etc. were all defined to begin the model. The specifications for each thruster were then noted with the thrust force, velocity, fuel mass, and mass when fuel is fully burned were all saved for each thruster. The rocket was defined to be starting from rest and with an initial mass of 2,951,000 kg which was including all the thrusters and fuel aboard the rocket ship. After all the initial parameters had been set the computational part of the model was developed.

The journey of the Saturn V rocket was separated into three main components. The use of the first thruster for takeoff and ascending through the Earth's atmosphere. The use of the second thruster to break through the Earth's atmosphere. The use of the third thruster to propel the rocket through space and hopefully into the moon's orbit. The model was setup so each thruster would fire till all the fuel had been depleted and then a jettison period of four seconds was allowed for the thruster to be detached and the following thruster to be ignited. The model was successfully developed and was able to model the rocket's altitude over time, the rocket's velocity over time, the thrust acceleration over time, and the rocket's fuel mass over time.

Once the model was successfully developed the model was tested to see if it was sufficiently accurate by varying Δt values. The Δt values were ranged from 25 to 0.1 with 0.1 being found to be sufficiently accurate for computational purposes. The graphs of the acceleration curves with various dt values can be seen in the figures below.

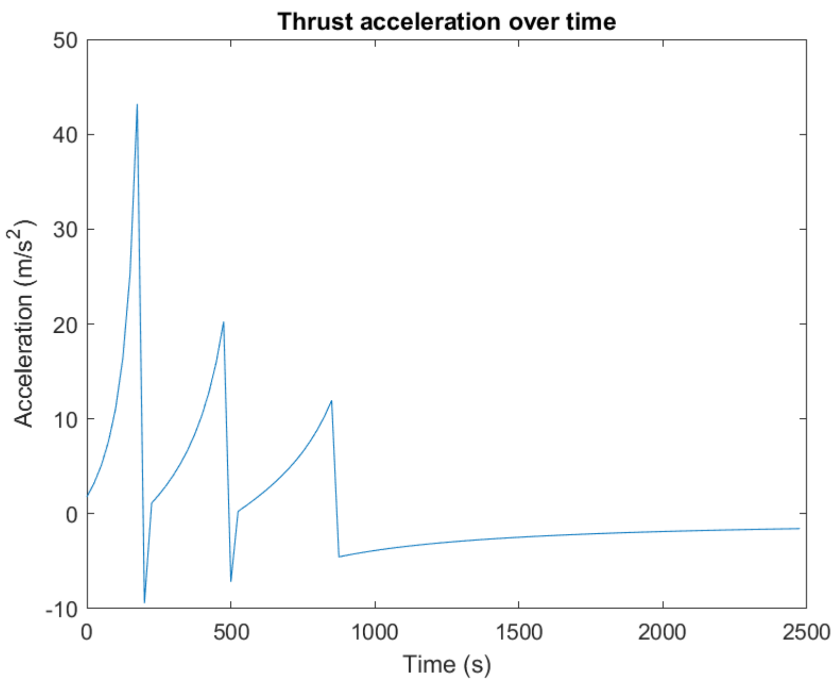

Figure X: Thrust acceleration over time with a dt of 25

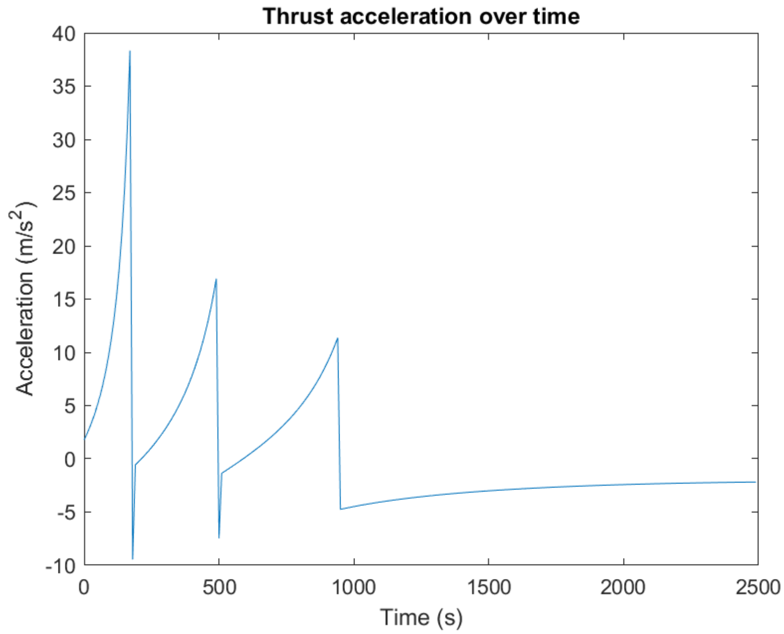

Figure X: Thrust acceleration over time with a dt of 10

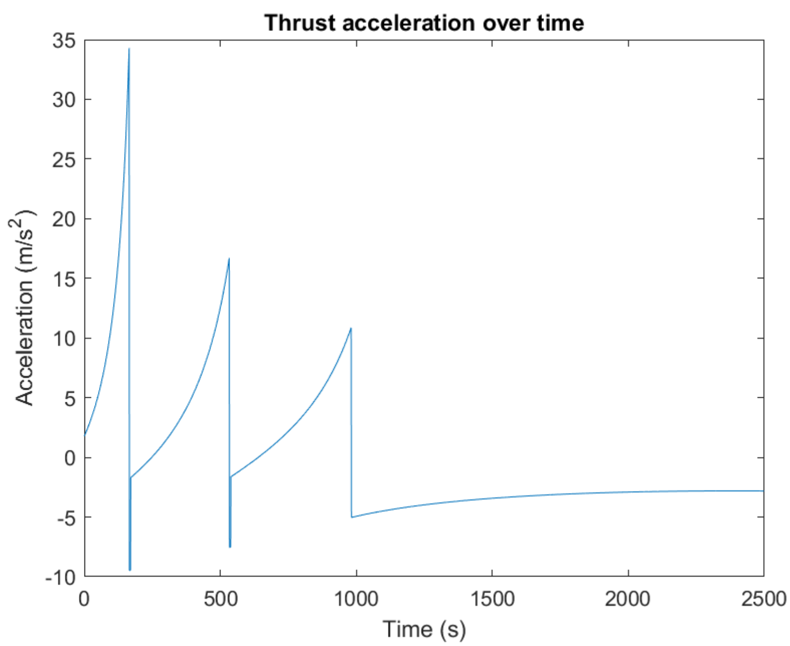

Figure X: Thrust acceleration over time with a dt of 1

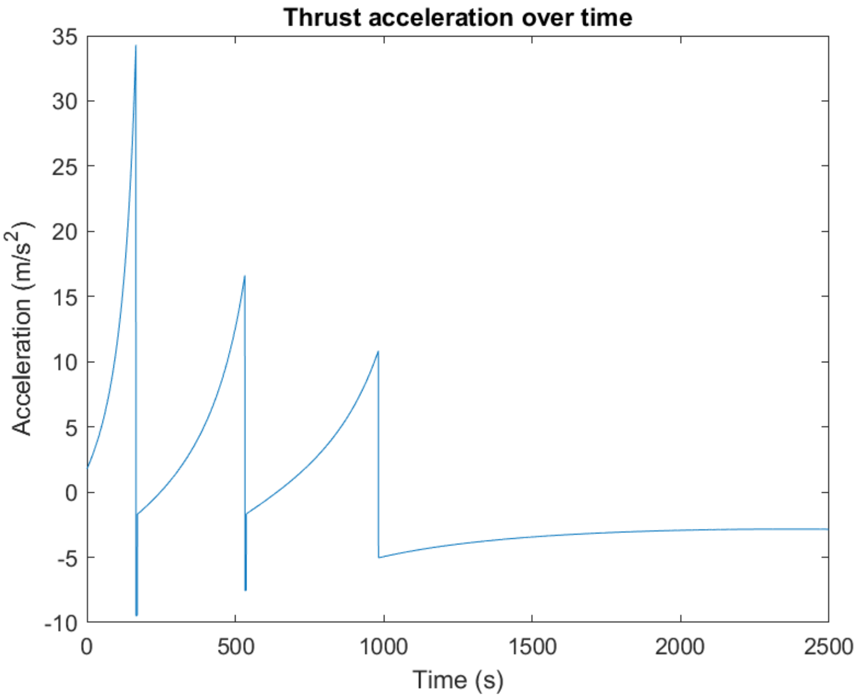

Figure X: Thrust acceleration over time with a dt of 0.5

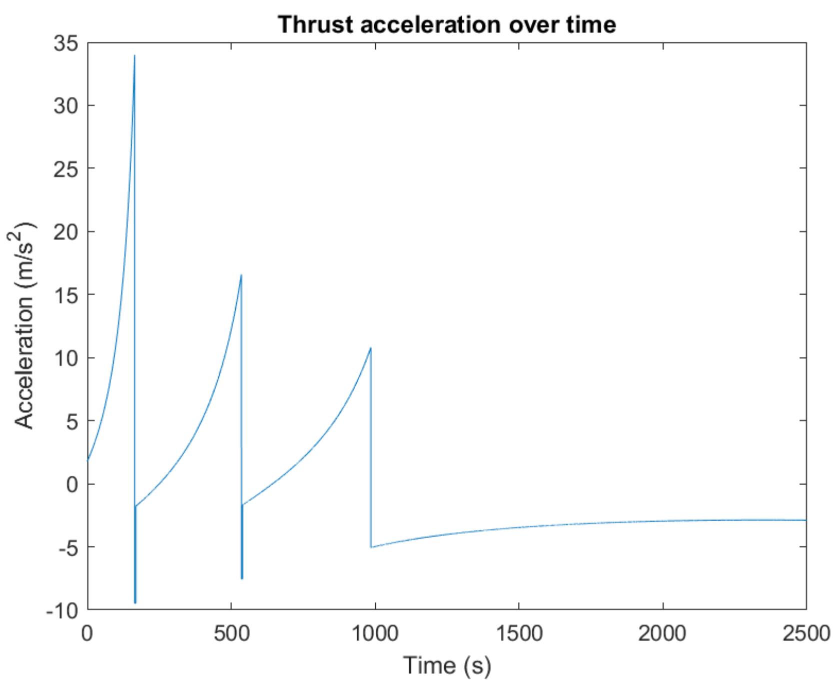

Figure X: Thrust acceleration over time with a dt of 0.1

Once the sufficiently small Δt value had been assessed, the model was deemed accurate and sufficient to begin analysis. The first analysis point was the altitude of the rocket, with a graph being developed and shown below in Figure X. It can be seen in the figure that the maximum altitude reached is 5,460,800 meters or 5468 km.

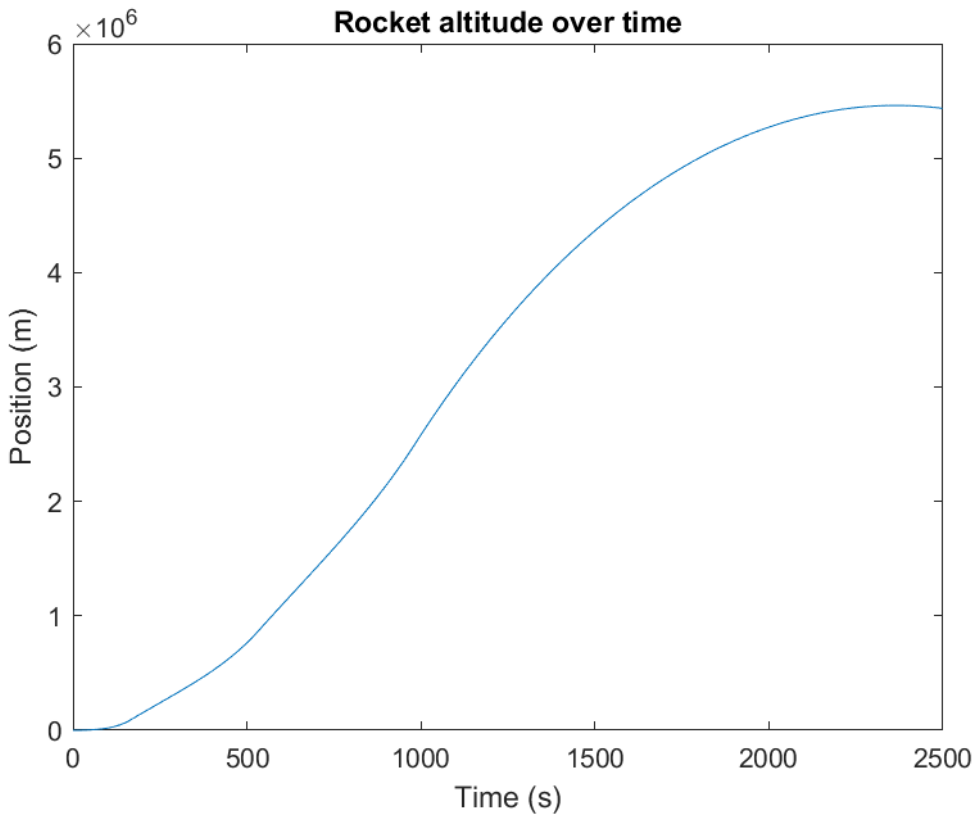

Figure X: Rocket altitude over time

The graph of the rocket's velocity was then plotted and can be seen below in Figure X. The velocity reaches a maximum of 4732.7 m/s at roughly 1000 seconds. This corresponds with the higher thrust values from the first and second thrusters as a higher velocity is needed to successfully break through Earth's atmosphere and successfully make it to space.

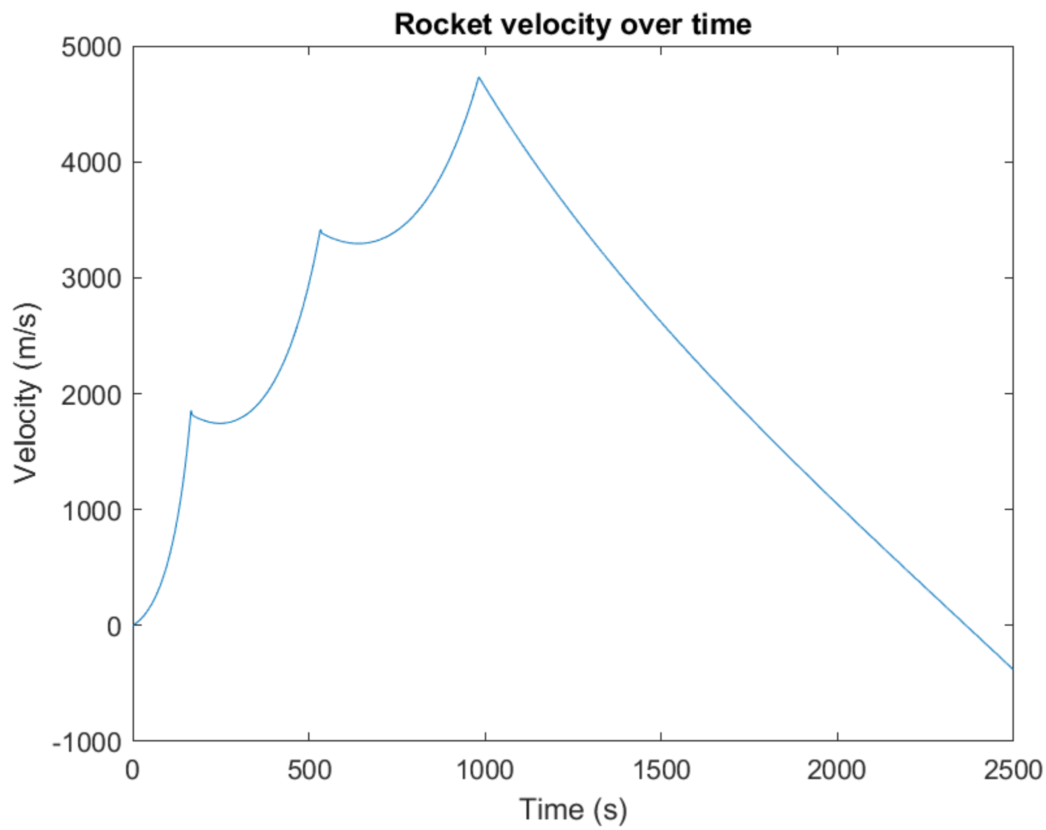

Figure X: Rocket velocity over time

The graph of the rocket's acceleration was then plotted and can be seen below in Figure X.

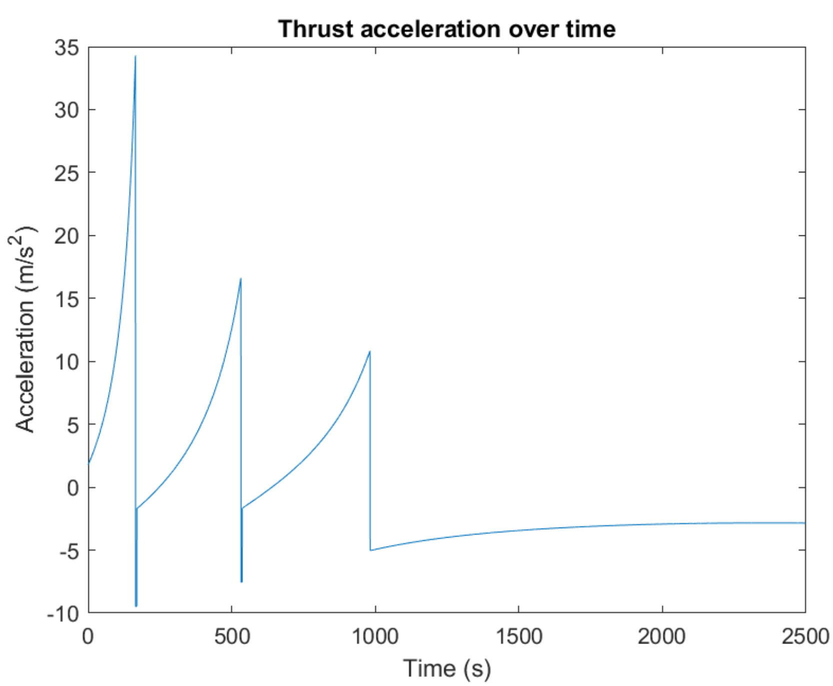

Figure X: Thrust acceleration over time

The thrust acceleration was broken down into three main segments corresponding with each thruster. The thrust acceleration was calculated using Eq. (2) for each thruster, with the thruster accelerating up to the point where all its respective fuel was burned. During the jettison period for each thruster, the rocket undergoes a brief period with a negative acceleration. This is due to the removal of the thrust component from Eq. (2) as the rocket experiences the gravitational force of the Earth pushing down on it until the following thruster is ignited. The first thruster provided the highest acceleration value of 34.2981 m/s^2, with each following thruster producing less thrust and less acceleration. This is because the most force is needed to move the object from rest and through the Earth's lower atmosphere. A function for the mass of the rocket was developed using Eq. (3) as the mass changes as the fuel burns and during the jettison period when the thruster whose fuel had been burned empty is dropped from the rocket. The function was divided into these two main components one with regards to while the fuel is burned and one with the removing of the thrusters. The graph of the function can be seen below in Figure X.

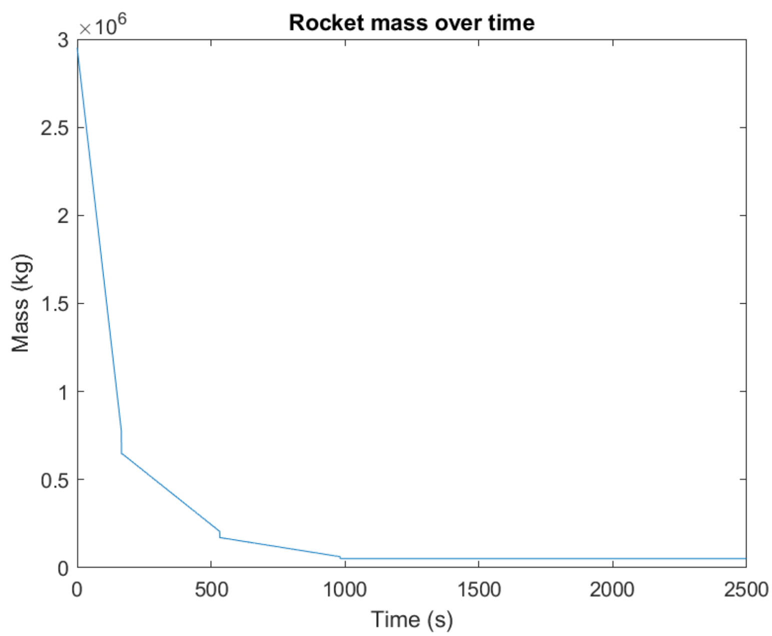

Figure X: Rocket mass over time

It can be seen that within the first five-hundred seconds, most of the fuel mass is consumed due to the required thrust needed to get the rocket through the Earth's lower atmosphere. It can also be observed in the figure at the five-hundred and one-thousand-second marks, where there is a linear decrease. This is from removing the thrusters whose fuel had just been consumed. The empty thrusters are dropped from the rocket as the thruster's fuel is consumed to make the rocket more efficient by removing the unneeded mass.

Now that the model has been fully developed and explained, it is time to analyze our Saturn V model compared to the real thing. It is known that the distance between the Moon and the Earth is roughly 384,403 km, and it is shown in our altitude graph that the maximum altitude reached by a rocket was 5,468 km. This is not even close to the distance required to reach the Moon, let alone travel outside Earth's radius, roughly 6371 km. Despite using all the relevant data from NASA, our model does not even come close to the Saturn V rocket's achievements. A couple of elements are missing from our model that prevents it from reaching this heroic feat of reaching the Moon. One of the most crucial elements is that the Saturn V rocket was not launched directly upward; instead, as it ascended into orbit, the rocket turned sideways into what is known as a "parking orbit." The rocket would be able to stay in this parking orbit for a couple of hours with no negative acceleration due to Earth's gravity during this period. In our model, there is no such period where the force of gravity is null; this is because our rocket is always pointing radially outward, making it always subject to the force of gravity. As the opposing force of gravity is not in play here, the fuel used can provide a much more meaningful acceleration from the fuel burned. As the rocket is ready to launch into the Moon's orbit, the thrusters would reignite and pick up more speed with the remaining fuel. The rocket is launched in a tangential path to its parking orbit while conserving all the initial velocity allowing it to reach the Moon's orbit successfully. In our model, all the fuel was burned before even entirely climbing out of the Earth's atmosphere due to the rocket being pointed radially outward the entire time, causing the major of the velocity to be stifled by the Earth's gravity.

Conclusions

The Saturn V computational model was determined to be sufficiently accurate with a dt value of 0.1, corresponding to Euler's calculating velocity and position method. Values of dt larger than this showed to be inaccurate for our computational purposes, while smaller values provided no noticeable increase in accuracy. The rocket in our model failed to reach the Moon, let alone even entirely pass the Earth's atmosphere with a peak altitude of 5,468 km. This was because the rocket is pointed radially outward the whole time, causing it to experience the entirety of Earth's gravitational force as it ascends through the atmosphere. The rocket would need significantly more thrust from the thrusters to reach the Moon under these circumstances. It would need to orbit the Earth horizontally to gain speed to break through the Earth's atmosphere and into the Moon's orbit.

Contributions

Zachary Abboud - Everything

Appendices

MATLAB code found below

```
clear;clc;
%define variables
dt=0.5;
G=6.67E-11;
M_E=5.97E24;
R_E=6.39E6;
U=2500;
N=round((U-1)/dt);

%preallocate matrices
time=zeros(1,N);
position=zeros(1,N);
velocity=zeros(1,N);
accel=zeros(1,N);
mass=zeros(1,N);


%defining thrust
thrust_1=34E6; %thrust in Newtons
Ve_1=2580; %velocity in m/s
MFS1=2169000; %mass of fuel
MSE1=782000; %rocket mass when stage 1 empty (131000+480000+119000+52,000)


thrust_2=5E6; %thrust in Newtons
Ve_2=4130; %velocity in m/s
MFS2=444000; %mass of fuel
MSE2=207000; %rocket mass when stage 2 empty (36,000+119,000+52,000)


thrust_3=1E6; %thrust in Newtons
Ve_3=4130; %velocity in m/s
```

```
MFS3=108000; %mass of fuel
MSE3=63000; %rocket mass when stage 3 empty (11,000+52,000)


%defining for index value of 1
time(1)=0;
position(1)=0;
velocity(1)=0;
mass(1)=2951000;
accel(1)=-(G*M_E)/((R_E+position(1))^2)+thrust_1/mass(1);


JettisonTime1=0;
JettisonTime2=0;
JettisonTime3=0;

for i=2:N
    if mass(i-1)>MSE1
        mass(i)=mass(i-1)-(thrust_1*dt)/Ve_1;
        position(i)=position(i-1)+velocity(i-1)*dt;
        velocity(i)=velocity(i-1)+accel(i-1)*dt;
        accel(i)=-(G*M_E)/((R_E+position(i))^2)+thrust_1/mass(i);
        JettisonStall1=mass(i)-131000;

    elseif mass(i-1)<MSE1 && MSE2<mass(i-1)
        if JettisonTime1<=4
            mass(i)=JettisonStall1;
            JettisonTime1=JettisonTime1+dt;
            position(i)=position(i-1)+velocity(i-1)*dt;
            velocity(i)=velocity(i-1)+accel(i-1)*dt;
            accel(i)=-(G*M_E)/((R_E+position(i))^2);
        else
            mass(i)=mass(i-1)-(thrust_2*dt)/Ve_2;
            position(i)=position(i-1)+velocity(i-1)*dt;
            velocity(i)=velocity(i-1)+accel(i-1)*dt;
            accel(i)=-(G*M_E)/((R_E+position(i))^2)+thrust_2/mass(i);
            JettisonStall2=mass(i)-36000;
        end

    elseif mass(i-1)<MSE2 && MSE3<mass(i-1)
        if JettisonTime2<=4
            mass(i)=JettisonStall2;
            JettisonTime2=JettisonTime2+dt;
            position(i)=position(i-1)+velocity(i-1)*dt;
            velocity(i)=velocity(i-1)+accel(i-1)*dt;
            accel(i)=-(G*M_E)/((R_E+position(i))^2);
        else
            mass(i)=mass(i-1)-(thrust_3*dt)/Ve_3;
            position(i)=position(i-1)+velocity(i-1)*dt;
            velocity(i)=velocity(i-1)+accel(i-1)*dt;
```

```
            accel(i)=-(G*M_E)/((R_E+position(i))^2)+thrust_3/mass(i);
            JettisonStall3=mass(i)-11000;
        end

    elseif mass(i-1)<MSE3
        if JettisonTime3<=4
            mass(i)=JettisonStall3;
            JettisonTime3=JettisonTime3+dt;
            position(i)=position(i-1)+velocity(i-1)*dt;
            velocity(i)=velocity(i-1)+accel(i-1)*dt;
            accel(i)=-(G*M_E)/((R_E+position(i))^2);
        else
            mass(i)=mass(i-1);
            position(i)=position(i-1)+velocity(i-1)*dt;
            velocity(i)=velocity(i-1)+accel(i-1)*dt;
            accel(i)=-(G*M_E)/((R_E+position(i))^2);
        end
    end
    time(i)=time(i-1)+dt;
end
%Plotting
plot(time,position)
title('Rocket altitude over time')
ylabel('Position (m)')
xlabel('Time (s)')
max(position)


plot(time,velocity)
title('Rocket velocity over time')
ylabel('Velocity (m/s)')
xlabel('Time (s)')
max(velocity)


plot(time,accel)
title('Thrust acceleration over time')
ylabel('Acceleration (m/s^2)')
xlabel('Time (s)')
max(accel)
plot(time,mass)
title('Rocket mass over time')
ylabel('Mass (kg)')
xlabel('Time (s)')
```